Re-Exam Subatomic Physics
Thursday, July 5 2012, 13:00-16:00

Gerco Onderwater

Rijksuniversiteit Groningen / KVI

Before you start, read the following:

- Write your name and student number on top of each page of your exam;
- Illegible writing will be graded as incorrect;
- Final grade = (10 + sum of points)/10;
- *Good luck!*

1 Allowed and Forbidden Processes (15 Points)

Two of the following reactions cannot occur (as far as we know) and a third cannot occur by the strong interaction. Find them and indicate the reason in each case. (5 points each)

Express each side of the equation in the constituent quarks. Eliminate like quarks and antiquarks. Strong interaction conserves charge, baryon number, quark flavor number, parity, *etc.*

(a) $K^- + p \to \bar{K}^0 + n$,
$\bar{u}s + uud \to \bar{d}s + udd$
$sud \to sud$: OK

(b) $\pi^+ + p \to K^+ + \Sigma^+$,
$\bar{d}u + uud \to \bar{s}u + uus$
$uuu \to uuu$: OK

(c) $\pi^- + p \to K^+ + \Sigma^0 + \pi^-$,
$\bar{u}d + uud \to \bar{s}u + uds + \bar{u}d$
$udd \to udd$: OK

(d) $\pi^- + p \to K^- + \Sigma^+$,
$\bar{u}d + uud \to \bar{u}s + uus$
$udd \to uss$: OK, but needs weak interaction to change flavor

(e) $\bar{K}^0 + p \to K^- + p + \pi^+$,
$\bar{d}s + uud \to \bar{u}s + uud + \bar{d}u$
$suu \to suu$: OK

(f) $\bar{p} + p \to \pi^+ + \pi^+ + \pi^- + \pi^- + \pi^+$,
Charge non-conservation : $q = 0$ in the initial state, $q = +1$ in the final state.

(g) $\pi^+ + p \to K^0 + \Sigma^0 + \pi^+ + K^+ + \bar{K}^0$,
$\bar{d}u + uud \to \bar{s}d + uds + \bar{d}u + \bar{s}u + \bar{d}s$
$uuu \to uuu$: OK

(h) $K^- + p \to \Sigma^+ + n + \pi^-$,
$\bar{u}s + uud \to uus + udd + \bar{u}d$
$sud \to uusddd$: violation of baryon number $B = 1 \to B = 2$

(i) $\pi^- + p \to \Sigma^+ + \Sigma^- + K^0 + \bar{p} + \bar{\Sigma}^+ + n$,
$\bar{u}d + uud \to uus + dds + \bar{s}d + \bar{u}\bar{u}\bar{d} + \bar{d}\bar{d}\bar{s} + udd$
$udd \to udd$: OK

(j) $\pi^- + p \to \bar{\Sigma}^- + \Sigma^0 + p$.
$\bar{u}d + uud \to \bar{u}\bar{u}\bar{s} + uds + uud$
$udd \to udd$

The notation $\bar{\Sigma}$ means the antiparticle to the Σ and the sign convention is that $\bar{\Sigma}^+$ has charge $+1$ and is therefore the antiparticle to the Σ^-.

2 Terminology (15 Points)

The terms *hadron*, *lepton*, *boson* and *fermion* are used in the classification of particles. Explain their meaning, giving examples of their use. Which can be applied to *quarks*?

hadron: A composite colorless particle that is subject to the strong interaction. Quarks, antiquarks and gluons are themselves thus not hadrons, but form the building blocks of them. Hadrons are cathegorized in baryons and mesons. Baryons consist of three quarks or three antiquarks. Mesons consist of a quark and an antiquark. More exotic combinations, glueballs, mesic molecules, pentaquarks, *etc.*, are also possible.

lepton: A particle that is not subject to the strong interaction. Leptons are either charged or neutral. Both a subject to the weak interaction, whereas the charged ones are also subject to the electromagnetic interaction. Charged leptons are the electron, muon and tau (and their antiparticles), the neutral ones are the corresponding neutrinos.

boson: A boson is a particle with integer spin, $S = 0, 1, 2, \cdots$. Elementary bosons are force carriers. The boson associated to the EM interaction is the photon, that of the strong force the gluon and of the weak interation the $\mathrm{W}^{\pm}$ and the Z. Composite particles with integer spins, *e.g.* a ^{4}He nucleus, are also bosons. Bosons obey Bose-Einstein statistics, meaning that there is no limit to the number of particles that may populate the same quantum-mechanical state.

fermion: Fermions are particles that have a half-integer spin, $S = 1/2, 3/2, 5/2, \cdots$. Elementary fermions make up matter. This are the leptons and quarks. Composite particle that consist of an odd number of fermions are also fermions themselves, *e.g.* the proton. Fermions obey the Fermi-Dirac statistics, *i.e.* only one particle may occupy a quantum state.

Quark: A quark is a fermion and because it has color stricktly speaking not a hadron. It is subject to all three fundamental interactions.

3 Cross sections (15 Points)

The TRIμP facility at KVI aims to study precisely nuclear β-decay. One of the isotopes of interest is $^{21}_{11}$Na. This isotope has a half-life of $T_{1/2} = 23\,\mathrm{s}$.

(a) We know there is only one stable Na isotope, which is ^{23}Na. What kind of β-decay is possible for ^{21}Na? (3 points)
Beta-plus decay. The number of neutrons is smaller than the number of protons. The most stable isotopes are located in the "valley of stability", which for small nuclei is characterized by the requirement that $N \simeq Z$. Hence a proton will be transformed into a neutron with the emission of a positron (to conserve charge) and an electron-neutrino (to conserve lepton number).

(b) Give the equation describing this decay. (3 points)
$^{21}_{11}\mathrm{Na} \rightarrow {}^{21}_{10}\mathrm{Ne} + e^+ + \nu_e$

(c) The production of the isotope happens by shooting a $^{20}_{10}$Ne beam at energy $E_b = 23\,\mathrm{MeV/u}$ (energy per nucleon of the beam particle) on a deuterium ($^2_1$H) gas target. Give the equation for this production reaction. (3 points)
$^{20}_{10}\mathrm{Ne} + {}^2_1\mathrm{H} \rightarrow {}^{22}_{11}\mathrm{Na}^* \rightarrow {}^{21}_{11}\mathrm{Na} + {}^1_0 n$

(d) The target is 10 cm long and it is kept at liquid nitrogen temperature (T=77.5K). The diatomic deuterium molecule can be assumed to be an ideal gas which at standard conditions (temperature $T_0 = 273\,\mathrm{K}$ and pressure $p_0 = 1\,\mathrm{bar}$) has a density of $6.023 \cdot 10^{23}$ particles per $22.4\,\mathrm{dm}^3$. What is the cross section for the production reaction, if one produces $^{21}\mathrm{Na}$ at a rate of $\Gamma_0 = 3 \cdot 10^8$ particles per second with a $^{20}\mathrm{Ne}^{6+}$ beam of 1 kW power (corresponding to $\Gamma_1 = 1.4 \cdot 10^{13}$ particles per second)? (6 points)
The scattering rate W is given by the relation

$$W = J \cdot N \cdot \sigma \tag{1}$$

with J the flux of incoming particles, N the density of scattering centers and σ the cross section. The luminosity is defined as $\mathcal{L} = JN$. Hence,

$$\sigma = \frac{W}{\mathcal{L}} \tag{2}$$

Here we have $W = \Gamma_0 = 3 \cdot 10^8/\mathrm{s}$. The density of scattering centers is

$$N = \rho_0 \frac{T_0}{T} L = 2 \cdot \frac{6.023 \cdot 10^{23}}{22.4}\,\mathrm{dm}^{-3} \cdot \frac{273\,\mathrm{K}}{77\,\mathrm{K}} \cdot 10\,\mathrm{cm} = 1.907 \cdot 10^{25}\,\mathrm{m}^{-2} = 1.907 \cdot 10^{-3}\,\mathrm{barn}^{-1} \tag{3}$$

Note the "2" to account for the fact that deuterium gas H_2 is diatomic. Also recall that $pV = nRT$ so that a gas a low temperature (and atmospheric pressure) has a higher density. So the cross section is

$$\sigma = \frac{3 \cdot 10^8\,\mathrm{s}^{-1}}{1.4 \cdot 10^{13}\,\mathrm{s}^{-1} \cdot 1.907 \cdot 10^{-3}\,\mathrm{barn}^{-1}} = 11.24\,\mathrm{mbarn} \tag{4}$$

4 Weak Interaction (15 Points)

For each of the following processes or phenomena, explain qualitatively why the weak interaction must be involved (3 points each):

(a) Beta momentum $\vec{p}_e$ asymmetry $A \neq 0$ in decay of polarized nuclei with spin $\vec{J}$: $\frac{d\sigma}{d\Omega} \propto 1 + A\vec{J} \cdot \vec{p}_e$
A non-zero value for this asymmetry signals a violation of parity. Parity violation only occurs in the weak interaction.

(b) atomic electron capture: $^7_4\mathrm{Be} + e^- \rightarrow {}^7_3\mathrm{Li} + \nu_e$
This reaction involves a neutrino, which only interacts via the weak interaction.

(c) Matter-Antimatter asymmetry in the universe
A distinction between matter and antimatter signals a violation of C and CP, which is only broken for the weak interaction. There is a possibility in the strong interaction via the $\bar{\theta}$-term.

(d) Kaon oscillations : $K^0 \leftrightarrow \bar{K}^0$
Kaon oscillations require flavor mixing. Mixing is possible when the mass and flavor eigenstates are not the same. This is only the case for the weak interaction and is parametrized in the CKM matrix.

(e) neutrino scattering: $\nu_\mu + e \rightarrow \nu_\mu + e$
This reaction involves neutrinos, which only interact via the weak interaction.

5 Quarks (15 Points)

The lowest mass (non-strange) baryons are:

	electric charge			
	-1	0	+1	+2
nucleons		n	p	
Δ-baryons	Δ^-	Δ^0	Δ^+	Δ^{++}

(a) What are the quark constituents of each of these states? (5 points)

n	udd
p	uud
Δ^-	ddd
Δ^0	udd
Δ^+	uud
Δ^{++}	uuu

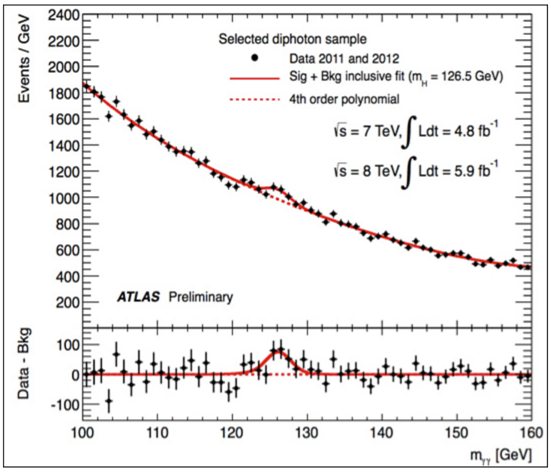
(b) Assuming that the quarks are in states of zero relative angular momentum, what fundamental difficulty appears to be associated with the Δ states, which have $J = 3/2$, and how is it resolved? (5 points)
The total spin of the Δ's is $3/2$. This includes the Δ^- and the Δ^{++}. They are composed of three quarks with identical flavor with their spin alinged. These are the lightest baryons, which have no angular momentum and no radial excitation. So with just the spin and spatial distribution there are tree fermions in a symmetric wavefunction, which violates the exclusion principle. This is resolved by introducing *color*, the strong interaction "charge", as a new degree of freedom. Quarks can have three color, red, green and blue. Using the color degrees of freedom the wavefunction can be antisymmetrized.

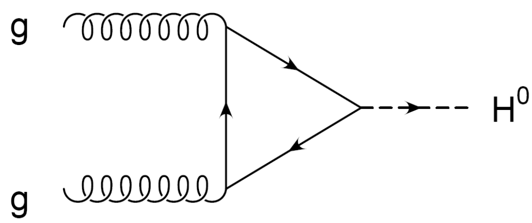
(c) The Δ^0 and the Λ (strangeness -1) both decay to a proton and π^--meson. Explain the large difference between the Δ^0 mean-life of $\sim 10^{-23}$ s and that of the Λ, which is 2.6×10^{-10} s. (5 points)
The Δ^0 can decay via the strong interaction, as no quark flavors change, $\Delta^0(udd) \rightarrow p(uud) + \pi^-(\bar{u}d)$. In the decay of the Λ a strange quark is transformed into an up quark; *i.e.* this decay is flavor changing, $\Lambda(uds) \rightarrow p(uud) + \pi^-(\bar{u}d)$. This decay is thus driven by the weak interaction, where flavor mixing is parametrized in the CKM

matrix, and specifically for the first generation by the Cabibbo angle. The first decay is Cabibbo allowed (proportional to the cosine of the Cabibbo angle), whereas the second is Cabbibo suppressed (proportional to the sine of the Cabibbo angle).

6 Higgs Boson (15 points)

Yesterday an announcement was made of the discovery of a new boson at CERN. This boson could be the long searched for Higgs boson. Results were presented for two final states that could occur when a Higgs boson decays: two photons (2γ; see figure below) and four leptons (4e, 4μ and $2e2\mu$).





1. Why was the conclusion that the observed bump belongs to a boson? Briefly explain your reasoning. (3 points)
   The two photon final state consists of two bosons itself, with $S = 1$. As angular momentum comes only in integer quanta, the total angular momentum of the two photons, and therefore of the particle that produced them, must be integer, hence it is a boson.

2. What can you conclude about the charge of the observed boson? Briefly explain your reasoning. (3 points)
   The two photon final state has no charge. Hence the boson must have charge zero. This is also possible for the four leptons final state, which thus consists of two leptons and two antileptons.

3. Explain how the mass of the decaying boson can be determined if you measure the momenta of the two photons. (3 points)
   In the rest frame of the decaying boson the two photons are emitted back-to-back and have an energy which corresponds to half the mass of the boson. Thus the invariant mass of the two photons is equal to the mass of the boson: $M_{boson} = M_{\gamma\gamma} = \sqrt{E_{tot}^2 - P_{tot}^2} = \sqrt{(E_1 + E_2)^2 - |\vec{p}_1 + \vec{p}_2|^2} = \sqrt{2(1 - \cos\theta)p_1 p_2}$. In the last step it was used that the photon is massless and that the energy and momentum are

thus equal. The angle θ and the momenta are measured in the lab frame. Measuring the photon momenta is thus sufficient to determine the boson mass.

4. The theoretically most important production mechanism of the Higgs boson is gluon fusion (see figure top right). Identify what kind of particle goes around in the triangle. (3 points)
   The particle in the triangle interacts with the gluons. Hence it must be a particle that is subject to strong interaction. This includes only the quarks. This particle in the triangle is thus a quark.

5. A Higgs boson could also decay into a quark-antiquark pair. Explain why the analysis of this decay channel is more complicated than that of two photons or 4 leptons (*hint*: what do you detect in this case?). (3 points)
   Quarks cannot be observed in isolation because they are not "white". Quarks therefore give rise to jets, *i.e.* sprays of hadrons. The production of a quark-antiquark pair may thus give rise to hundreds of particles that all together have an invariant mass equal to that of the decaying boson.

Constants

Speed of light	c	$2.998 \cdot 10^{8}$	m/s
Planck constant	h	$4.136 \cdot 10^{-24}$	GeV·s
	$\hbar = \frac{h}{2\pi}$	$6.582 \cdot 10^{-25}$	GeV/c
Electron charge	e	$1.602 \cdot 10^{-19}$	C
Electron mass	m_e	0.510998918(44)	MeV/c^2
Proton mass	m_p	938.272029(80)	MeV/c^2
Neutron mass	m_n	939.565360(81)	MeV/c^2
Deuteron mass	m_d	1875.61282(16)	MeV/c^2
Alpha particle mass	m_α	3727.37917(32)	MeV/c^2
Electron neutrino mass	m_{ν_e}	< 2.2	eV/c^2
Muon mass	m_μ	105.658369(9)	MeV/c^2
Tau mass	m_τ	1776.84(17)	MeV/c^2
Charged pion mass	$m_{\pi^\pm}$	139.57018(35)	MeV/c^2
Neutral pion mass	m_{π^0}	134.9766(6)	MeV/c^2
$W^\pm$-boson mass	m_W	80.403(29)	MeV/c^2
Z^0-boson mass	m_W	91.1876(21)	MeV/c^2
Avogadro's number	N_A	$6.02214179(30) \cdot 10^{23}$	mol^{-1}

Semi-Emperical Mass Formula (Bethe-Weizsäcker)

$$M(A,Z) = Nm_n + Zm_p - a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_a \frac{(A-2Z)^2}{4A} + \frac{\delta}{A^{1/2}}$$

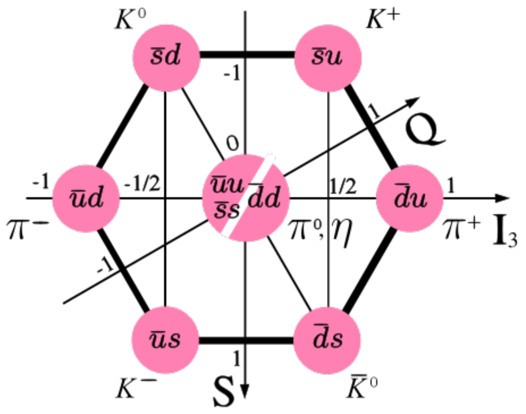
$$\begin{aligned} a_v &= 15.67 \ \text{MeV}/c^2 \\ a_s &= 17.23 \ \text{MeV}/c^2 \\ a_c &= 0.714 \ \text{MeV}/c^2 \\ a_a &= 93.15 \ \text{MeV}/c^2 \\ \delta &= 0 \ \text{odd } A \\ &= -11.2 \ \text{MeV}/c^2, \quad Z \text{ and } N \text{ even} \\ &= +11.2 \ \text{MeV}/c^2, \quad Z \text{ and } N \text{ odd} \end{aligned}$$

Conversion Factors

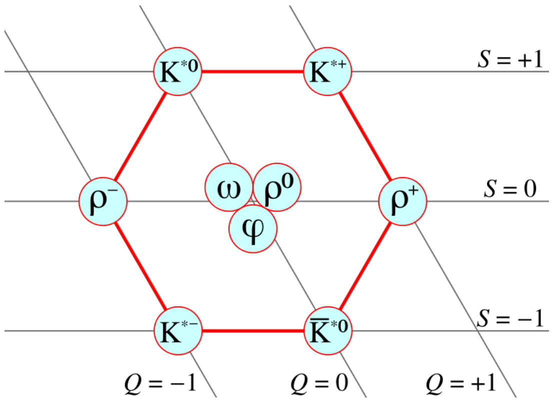
Electronvolt	eV	$1.60217653(14) \cdot 10^{-19}$	J
Tesla	T	$0.561 \cdot 1030$	MeV/c^2·C·s
kilogram	kg	$5.60958896(48) \cdot 10^{35}$	eV/c^2
barn	b	$1 \cdot 10^{-28}$	m^2

Note: For some of the questions different approaches are possible, such that you may not necessarily need all of the given constants and equations. Unless specifically stated, the final results are sufficient if given to 2 significant figures (2 leading digits).

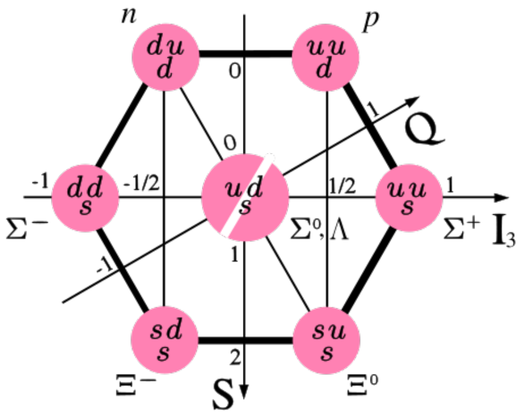
Baryon and Meson Composition



Spin-0 Mesons



Spin-1 Mesons



Spin-1/2 Baryons

name	composition	mass [GeV/c^2]
J/ψ	$c\bar{c}$	3097
D^+	$c\bar{d}$	1869
D^0	$c\bar{u}$	1864
$\bar{D}^0$	$\bar{c}u$	1864
D^-	$\bar{c}d$	1869

Isotope Table near ^{20}Na

	7		9		11		13		Z
1									
3	10N								
	11N	12O							
5	12N	13O	14F						
	13N	14O	15F	16Ne					
7	14N	15O	16F	17Ne	18Na	19Mg			
	15N	16O	17F	18Ne	19Na	20Mg	21Al	22Si	
9	16N	17O	18F	19Ne	20Na	21Mg	22Al	23Si	24P
	17N	18O	19F	20Ne	21Na	22Mg	23Al	24Si	25P
11	18N	19O	20F	21Ne	22Na	23Mg	24Al	25Si	26P
	19N	20O	21F	22Ne	23Na	24Mg	25Al	26Si	27P
13	20N	21O	22F	23Ne	24Na	25Mg	26Al	27Si	28P
	21N	22O	23F	24Ne	25Na	26Mg	27Al	28Si	29P
15	22N	23O	24F	25Ne	26Na	27Mg	28Al	29Si	30P
	23N	24O	25F	26Ne	27Na	28Mg	29Al	30Si	31P
N	24N	25O	26F	27Ne	28Na	29Mg	30Al	31Si	32P